A Brain Computer Interface Controlled, 3-D Printed Prosthetic Hand

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# Abstract

Millions of people across the world are living with a form of upper limb paralysis. Studies have shown that the most prominent causes of this paralysis are stroke, spinal cord injury, and multiple sclerosis (MS) < <https://www.christopherreeve.org/living-with-paralysis/stats-about-paralysis>>. Paralysis of the upper limbs can be caused by a breakdown in the neural signalling pathway, between the motor cortex and the skeletal muscles in the paralysed limb. In many cases, the function of the cortex remains sufficiently intact for the neural signalling patterns to be intercepted, decoded, and an appropriate motor action executed by an assistive device. This is the role of Brain Computer Interface (BCI) Neuroprosthesis. Such devices have the potential to make a dramatic positive impact on the lives of millions of people living with paralysis or reduced motor function, restoring ability and independence. This project will explore the design and development of a 1 Degree of Freedom (DOF) prosthetic hand, controlled by the real-time, online classification of electroencephalography (EEG) signals obtained from the scalp of a healthy 21-year-old male. To achieve this outcome, hardware and software design and implementation, filament deposition manufacturing (FDM), simulation, data-acquisition, machine learning, control theory, and signal processing were conducted. This report will provide the biological background behind voluntary movement in the context of a BCI system, a summary of non-invasive BCI systems published in literature, a simulation of published feature extraction and classification techniques, an outline of the designed system and methods, experimental results, conclusions and recommendations for future research in this area. <RESULTS>

# Acknowledgements

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# Nomenclature

# Introduction

# Biological Background

The human brain is an incredible complex entanglement of various specialisations of cells. The modern field of neuroscience can be traced back to the 19th century, when Jean Legallois concluded that the medulla oblongata had a dominant role in respiration < <https://onlinelibrary.wiley.com/doi/abs/10.1002/clc.4960181015>>. Since then, while unprecedented progression and understanding has occurred, there are still many neurological processes, such as memory and consciousness, with unknown mechanisms. Recently, advanced studies in the physical structure of the brain, connectomics, aims to increase humanity’s understanding by examining the connectedness of functional structures of the brain. Additionally, if the neural connections are known to a sufficient degree, whole regions of the brain may be computationally simulated < <https://www.thevirtualbrain.org/tvb/zwei/neuroscience-simulation>>. This would provide a testbed for novel neuroactive medicines, or predictions of the impact of a stroke in a particular brain region.

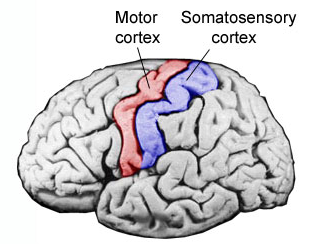
## Neurons

The general term for the cells that make up the brain are neurons. A neuron consists of dendrites (inputs), a soma, an axon, and axon terminals (outputs) < <https://qbi.uq.edu.au/brain/brain-anatomy/what-neuron>>. The neurons that make up the brain communicate through electrochemical signalling, called action potentials (APs). The cell membrane of the neuron sits at a resting potential of approximately -70 mV <<https://courses.lumenlearning.com/wm-biology2/chapter/resting-membrane-potential/>>. The membrane potential is set by the concentration of different ions inside the intracellular fluid of the soma. The intracellular fluid contains a higher concentration of potassium ions ([K+]) and a lower concentration of sodium ions ([Na2+]) than the intercellular fluid. The concentration gradient of the sodium ions is greater than that of the potassium ions, due to an increased permeability of the membrane to potassium < <https://www.khanacademy.org/science/biology/human-biology/neuron-nervous-system/a/the-membrane-potential>>.

Received APs from neighbouring neurons raises the membrane potential of the neuron incrementally, until a threshold potential is reached, when a series of events occur. Initially, voltage gated sodium channels open, allowing an influx of Na2+ ions from the intercellular fluid to enter the cell, depolarising it further. Once the membrane potential is slightly more positive than the intercellular fluid, voltage gated potassium channels open, allowing for K+ efflux from the cell body, and the voltage gated sodium channels close. This allows the cell to depolarise again. When the cell returns to its resting potential, the voltage gated potassium channels close < <https://opentextbc.ca/anatomyandphysiology/chapter/12-4-the-action-potential/>>.

APs initiate in the soma, then propagate along the axon to the axon terminals, where the neuron may synapse (connect) to a neighbouring neuron < <https://qbi.uq.edu.au/brain-basics/brain/brain-physiology/action-potentials-and-synapses>>. After passing through various cortical areas, the signal may synapse to a projection neuron, which will carry it down the spinal cord and into the peripheral nervous system.

## Cortical Areas for Voluntary Movement

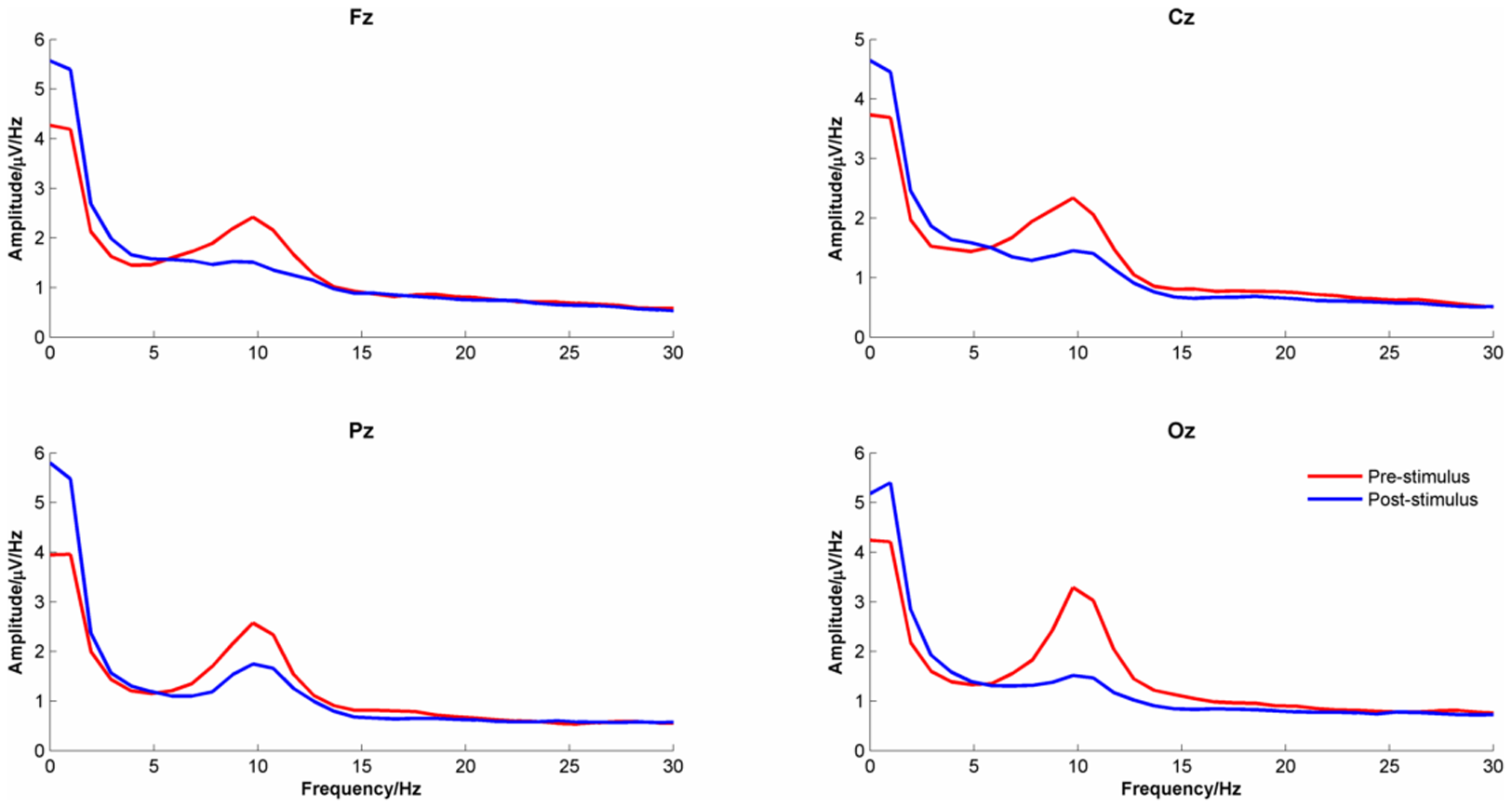
The brain has been divided into defined cortical areas, each with a specific set of connections and functions. The cortical area responsible for voluntary motor control is the motor cortex (M1) <<https://brainconnection.brainhq.com/2013/03/05/the-anatomy-of-movement/>>, located in the precentral gyrus, and shown in <Figure>.

<<https://operativeneurosurgery.com/doku.php?id=motor_cortex>>

The M1 is present in the left and right hemispheres of the brain. Left side movements are lateralised to the right hemisphere M1, while right side movements are lateralised to the left hemisphere M1, thus the M1 possesses contralateral control <<https://nba.uth.tmc.edu/neuroscience/m/s3/chapter03.html>>.

The imagination or execution of a motor control command (e.g. right-hand grasp) will case an Event Related Desynchronisation (ERD) or an Event Related Synchronisation (ERS) <<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4077682/>>. During an ERD, neurons which were typically firing APs at a similar frequency will begin to fire APs at differing frequencies, resulting in a decrease in the spectral power of the old, synchronous frequency. Contrarily, during an ERS, neurons which were typically firing APs at different frequencies will begin to fire APs at a similar frequency, resulting in an increase in the spectral power at the new, synchronous frequency <<https://neupsykey.com/eeg-event-related-desynchronization-erd-and-event-related-synchronization-ers/>>. Many BCI systems classify motor intent by considering the change in spectral power characteristic to an ERD/ERS. <<https://www.ncbi.nlm.nih.gov/pubmed/23366538>> <<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5191031/>>

<<https://link.springer.com/chapter/10.1007/978-3-319-68619-6_27>>.



<<https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0045630>>

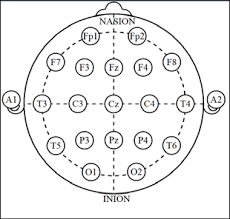
## 3. Electroencephalography (EEG)

EEG is a low-cost, non-invasive recording method to measure neural activity. It relies on the electrical currents generated by populations of thousands of neurons firing action potentials to conduct a small voltage signal to the skin of the scalp. The signals are detected using an array of electrodes.

Since EEG is measuring the activity of a general population of neurons, the spatial resolution of EEG is low. It is difficult to distinguish activity originating directly under the EEG electrode from activity emanating from a cortical region under an adjacent electrode <<https://www.electronicsandcommunications.com/2018/08/advantages-disadvantages-applications-of-eeg.html>>. However, EEG systems offer a high temporal resolution, limited only by the sampling rate of the data acquisition system < <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4548479/>>. This makes EEG neural recording methodologies attractive in a time-frequency Brain Computer Interface environment.

The electrodes commonly used to record the scalp potentials are classified as “wet” or “dry”. Wet electrodes require extensive scalp preparation, and the application of a conductive gel < <https://www.cambridge.org/core/journals/acta-neuropsychiatrica/article/techniques-for-effective-eeg-subject-preparation/27A5B6EF26DEE8EF5A714C51E2EF8E60>>. This made wet electrodes cumbersome, and slowed the testing process. Recent advances in dry electrodes have improved their signal acquisition properties to perform similarly to wet electrodes <<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4168519/>> <<https://www.sciencedirect.com/science/article/pii/S1053811918307961>> <<https://www.ncbi.nlm.nih.gov/pubmed/28000254>>. Dry electrodes do not require extensive preparation of the scalp, and can perform effectively through hair, due to the spiked nature of the electrode. <FIGURE> compares the visual characteristics of wet and dry electrodes <<https://www.fri-fl-shop.com/product/new-longer-5mm-spike-disposable-reusable-dry-eeg-electrode-tde-210/>> <<https://neuro.natus.com/products-services/grass-reusable-stamped-cup-eeg-electrodes>>.



The international standard EEG electrode locations follow the 10-20 system, presented in <FIGURE>. A standardised approach to electrode location ensures the same cortical areas are measured across trials and subjects. Additionally, this approach enables hardware to be manufactured to suit the standard, rather than follow proprietary location schemes. Wang et. al, identify the optimal locations for EEG electrodes to record motor imagery are C3, C4, and FCz on the 10-20 scheme, as these locations correspond to the right hand, left hand, and sensory motor area respectively < <https://www.researchgate.net/publication/227176331_Practical_Designs_of_Brain-Computer_Interfaces_Based_on_the_Modulation_of_EEG_Rhythms>>.

# Literature Review

To facilitate an online Brain Computer Interface control system, it is widely accepted that three main functional blocks are required. These blocks are:

1. Data Acquisition and Pre-processing,
2. Feature Extraction, and
3. Classification.

Each of these blocks can consist of different approaches and procedures. A literature review, in conjunction with MATLAB simulations, was conducted to predict the relative performance of each potential system. This enabled a reduction in the number of potential solutions, and the eventual derivation of an optimal system.

## Data Acquisition and Pre-processing

To analyse the neurological changes initiated by motor imagery processes, a method of effectively capturing the electrical signalling of the brain is required. This method should minimise the effect of noise from external sources. Then, since the device is a Brain Computer Interface, a method of digitising, communicating, and recording the results is necessary. Following this, some pre-processing may be necessary.

### Signal Recording Methods

There are several well documented means of recording neural activity from the human brain. At a high level, these can be divided into invasive and non-invasive methods. Invasive methods, such as electrocorticography (ECoG), offer some advantages over non-invasive methods. These methods exhibit reduced signal attenuation due to the sensors being placed under the parietal bones of the skull < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/4360075>>, improving the signal to noise ratio. They are also not susceptible to artefacts introduced from oculomotor or skeletal motor events. However, placement of these subdural electrodes poses significant risk, and is usually only planted for clinical reasons in patients for epilepsy monitoring < <https://med.nyu.edu/thesenlab/research-0/intracranial-eeg/>>. This risk is so significant, that a survey conducted by Engdahl et al. concluded that only 39% of participants indicated an interest in prosthesis controlled by (implanted) cortical interfaces, compared to 83% for myoelectric control <<https://link.springer.com/article/10.1186/s12984-015-0044-2>>. Additionally, the scope of this report restricts the final system to be low-cost, which precludes an invasive recording method.

Scalp-based electroencephalography (EEG) is a common low-cost, portable recording method <<https://imotions.com/blog/eeg-vs-mri-vs-fmri-differences/>>. Using this approach, electrodes are placed on the scalp of the subject, which detect the ionic currents generated by the brain during an action potential <<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4168519/>>. These signals are extremely small, and are prone to radiated EMI from proximal sources. For decades, the accepted standard for electrodes were Ag/AgCl wet electrodes. These devices were passive, and required extensive preparation of the skin and electrode to ensure a low electrode-skin impedance. Usually this preparation is conducted by a trained specialist, took several minutes, and left a gel residue on the subject’s head <<https://www.sciencedirect.com/science/article/pii/S1053811918307961>>. Recently, however, new technology in the form of an active, dry electrode has enabled rapid acquisition of EEG signals, requiring practically no setup. These dry electrodes have been evaluated by multiple sources to perform to an equivalent standard as wet electrodes <<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4168519/>> <<https://www.sciencedirect.com/science/article/pii/S1053811918307961>> <<https://www.ncbi.nlm.nih.gov/pubmed/28000254>>. Such devices are attractive in this project, as they will enable rapid testing and development, at a low cost.

### Digitalisation Methods

Due to the signal’s susceptance to EMI, and to reduce the impact of aliasing introduced by time-domain sampling, most systems proposed in literature have included some form of lowpass filtering on the input stage of the analog to digital converter (ADC) <<https://ieeexplore.ieee.org/abstract/document/7391321>> <<https://ieeexplore.ieee.org/document/7359346>>. Application notes for some EEG analog front end (AFE) integrated circuits indicate that a single order passive RC lowpass filter is sufficient to mitigate the coupling of high frequency noise sources, and reduce the amplitude of high frequency content of the signal, in turn reducing the effect of aliasing <<http://www.ti.com/lit/ug/slau443b/slau443b.pdf>>. This is an attractive solution, since these filters consume no power, enabling the data acquisition front end to be powered from a DC battery. Powering the device in such a manner reduces the likelihood of introducing condition EMI into the signal from a rectified source <<https://www.maximintegrated.com/en/app-notes/index.mvp/id/653>>.

Published literature appreciates that designing an AFE from scratch is a non-trivial task <<https://pdfs.semanticscholar.org/99e1/47917ea94e804a7c021bcacb8f58ed8dd3ed.pdf>>, and many papers suggest using an off the shelf component <<https://pdfs.semanticscholar.org/dcfc/deeffdfcac12d5b57781b52f12259f82073e.pdf?_ga=2.118223119.767518773.1566868245-1146442152.1566868245>>. Several studies have selected the Texas Instruments (TI) ADS1299 chip to serve as the AFE when designing a BCI <<https://pdfs.semanticscholar.org/dcfc/deeffdfcac12d5b57781b52f12259f82073e.pdf?_ga=2.118223119.767518773.1566868245-1146442152.1566868245>> <<https://pdfs.semanticscholar.org/99e1/47917ea94e804a7c021bcacb8f58ed8dd3ed.pdf>> <<https://dl-acm-org.ezproxy.newcastle.edu.au/citation.cfm?doid=2990299.2990304>> <<https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/7527437?arnumber=7527437&tag=1>> <<https://link-springer-com.ezproxy.newcastle.edu.au/chapter/10.1007%2F978-981-10-4361-1_106>> <<https://www.semanticscholar.org/paper/Wearable-Bluetooth-Brain-Computer-Interface-for-and-Wild-Pegan/bae82d84b4384bd86cf942139ac9a45326f3083d>>. This chip is an attractive solution, due to its 24-bit resolution, high sample rate range (from 250 to 16k samples per second), high common-mode rejection ratio (-110 dB), low input referred noise (1 µVpp over a 70Hz bandwidth), and small, self-contained package <<http://www.ti.com/product/ADS1299>>. The performance of this chip exceeds the performance of the AFE designed by Hu et al. in <<https://pdfs.semanticscholar.org/99e1/47917ea94e804a7c021bcacb8f58ed8dd3ed.pdf>>. The chip interfaces to a microcontroller through an SPI interface. Additionally, the ADS1299 can be configured to use a single reference electrode, in a referred montage scenario, or establish differential signals when recording in a differential montage <<http://www.ti.com/lit/ds/symlink/ads1299.pdf>>. An evaluation board is available for this chip, however, TI has published a recommended schematic and PCB layout, which may serve as a valuable starting point from which a custom front end can be designed <<http://www.ti.com/lit/ug/slau443b/slau443b.pdf>>. Thus, due to extensive prior works, and detailed supporting information, the TI ADS1299 AFE may be a viable solution to the EEG digitalisation problem.

## Feature Extraction

To reduce the computational intensity required to classify the EEG event, a feature extraction algorithm is applied to the acquired signal. Proposed algorithms include a Short Time Fourier Transform approach (STFT), an Adaptive Autoregressive Model (AAR), and a Discrete Wavelet Transform (DWT) approach. Additionally, a hybrid approach consisting of an amalgamation of Multivariate Empirical Mode Decomposition (MEMD) and STFT is considered. It is noted that the articles cited use different testing data and methodologies, thus a valid absolute performance comparison cannot be made without evaluating these approaches against the same dataset. Additionally, the quoted performance of the feature extraction algorithm is dependent on the performance of the classification technique employed by the study, again reducing the validity of an absolute performance comparison.

### Adaptive Autoregressive Model (AAR)

An Adaptive Autoregressive Model (AAR) is a variation on an Autoregressive Model (AR), which allows the parameters of the model to adjust over time. This feature is beneficial when applied to an EEG signal, as the signal is non-stationary <REFERENCE NEEDED>. An AAR model of order is described by:

, and

Where is the n-th sample of the EEG observation, is the zero-mean-Gaussian noise with variance , and are the time-varying AR coefficients. Thus, a sample is predicted by the past samples, with new information introduced through the model through the innovation function , which in the above example is a normal distribution < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/6740323>>.

In some models, it is not beneficial to apply any equal weighting to all historical values. In this case, a “forgetting factor” is introduced, where:

which usually takes the form .

As proven by Haykin in < <https://books.google.com.au/books/about/Adaptive_Filter_Theory.html?id=eMcZAQAAIAAJ&redir_esc=y>>, a recursive algorithm is applied to find the vector such as to minimise the sum of the error squares:

where is the vector of desired responses.

Applying the following variable changes:

enables the criterion presented in <EQUATION> to be rewritten as the standard least squares criterion:

Thus, assuming a causal signal, the least squares solution can be obtained as:

where:

It can be shown through the matrix inversion formula, that:

can be written as:

Defining and

it can be shown that:

Thus, it is now possible to derive a time-update equation for :

where

Now all the required equations to form the recursive least squares algorithm have been produced, and are summarised as follows:

It is shown by Haykin that this recursive algorithm can be solved with relative computational ease, however the computational intensity increases with < <https://books.google.com.au/books/about/Adaptive_Filter_Theory.html?id=eMcZAQAAIAAJ&redir_esc=y>>.

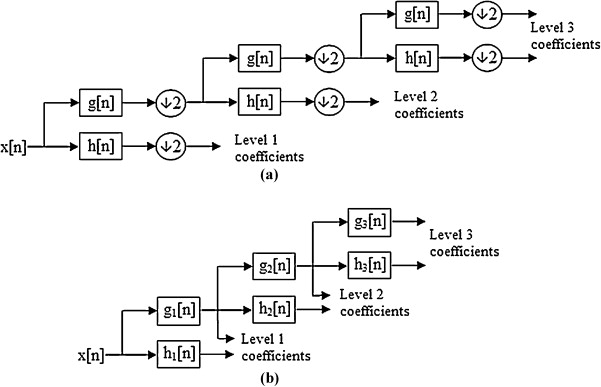
Such a model considers only time-domain parameters, thus utilising the high temporal resolution offered by EEG data acquisition systems at the peril of ignoring the prominent frequency-domain characteristics induced by ERPs. Low order models () are not computationally intensive, however accuracy increases with model order. Prior works indicate that AAR approaches may result in low accuracy, with Rodríguez-Bermúdez & García-Laencina, 2012 reporting a 62.2% accuracy <<https://link.springer.com/article/10.1007/s10916-012-9893-4>>.

### Discrete Wavelet Transform (DWT)

The Discrete Wavelet Transform (DWT) is a time-frequency approach, thus utilising both the temporal resolution of EEG signals, and the frequency characteristics of ERPs. This approach involves passing the signal through a multi-stage filter bank, consisting of highpass and lowpass filters. The highpass filtered signal produces the “detail coefficients” at that level of the filter bank, while the lowpass filtered signal produces the “estimate coefficients”. The lowpass filtered signal is then passed to the next level of the filter bank, as shown in <FIGURE>.

Since the frequency content of the signal has been bandlimited to one half of its original bandwidth, by Nyquist’s theorem, the signal can be subsampled by a factor of 2, reducing the number of samples at each step. In turn, this doubles the frequency resolution. Thus, DWT approaches offer high spectral resolution over the low frequency content. This characteristic is beneficial when extracting features from EEG data for motor imagery, since these signals occur in the low frequency spectrum, however a higher resolution will increase the computation time compared to analysing the STFT over a similar bandwidth.

When paired with a Long-Term Short-Term Memory (LSTM) classifier, Jie Zhou et al. achieved an accuracy of 91.43% on the Graz 2003 Brain Computer Interface Competition III dataset <<https://ieeexplore.ieee.org/document/8408108>>. Thus, the DWT method may be suitable for online motor imagery feature extraction.



< <https://www.sciencedirect.com/science/article/pii/S2314717214000075>>

### Short Time Fourier Transform (STFT)

The Short Time Fourier Transform (STFT) approach is another time-frequency feature extraction method, again making use of the high temporal resolution offered by EEG signals and the identifiable frequency characteristics of ERPs. Under this approach, a sliding feature extraction window of defined length is passed over the EEG signal (in online analysis, this can be implemented using a FIFO buffer), and the discrete Fourier transform (DFT) of the signal in the window is taken. There are several windows to choose from, including:

1. Rectangular. This is the least computationally intensive window, however leads to increased sidelobes in the frequency domain which will need to be filtered.
2. Triangular. This window is slightly more computationally intensive, however the sidelobes will be somewhat compressed.
3. Hanning. This window is more computationally intensive, however offers the best compression of frequency domain sidelobes.

Huang et al. tested rectangular, triangular, Hamming, and Hanning windows when extracting features from Steady State Visual Evoked Potentials (SSVEP). Each of the windowed signals were passed to the same Linear Discriminant Analysis (LDA) classifier. It was found that if no highpass filter was applied, the triangular, Hamming, and Hanning windows performed equivalently (classification accuracy was within 0.5%), while the rectangular window did not perform as well. However, after a 4th order Butterworth highpass filter (cut-off frequency of 2 Hz) was applied to all the windowed signals, the rectangular window outperformed the rest of the windows. Thus, it is recommended by the authors of that paper that a rectangular window be used in conjunction with a 4th order highpass filter, for SSVEP analysis < <https://link.springer.com/content/pdf/10.1007%2F978-3-642-25489-5.pdf>>. However, if a 4th order highpass filter is not applied, a triangular window is a computationally efficient, high-performance window function.

To circumvent the amplitude suppression introduced at the fringes of the window, the feature extraction window is “slid” over the length of the EEG signal. This means that each time domain sample of the signal (except those within one half of the window length from the extremities of the signal) spend equal amounts of time at the centre of the window.

Coyle et al. implemented a STFT approach to extract features from left and right hand movement EEG signals. The features were then classified by an LDA classifier. This study produced an average classification accuracy of 88.425% across 4 trials on 3 subjects < <https://link.springer.com/content/pdf/10.1155/ASP.2005.3141.pdf>>. Thus, a STFT approach to feature extraction may be a viable solution to online BCI.

### Discrete Cosine Transform (DCT)

The Discrete Cosine Transform (DCT) was developed by Chen and Pratt in 1984, and is similar to the DFT, however carries some advantages. Rather than decomposing the signal into a summation of sine and cosine functions (represented as complex exponentials), the DCT uses only cosines, and it thus completely real-valued. This reduces computational complexity < <https://pdfs.semanticscholar.org/d323/4ff6ac47a2284b629f92973344b67657fa0c.pdf>>. The DCT coefficients, , for an -point signal, , can be computed by <EQUATION>.

<<http://homepages.cae.wisc.edu/~ece554/website/Xilinx/app_notes/DCT_IDCT%20Customer%20Tutorial%20custdct.pdf>>. Additionally, through applying a zonal coding strategy, the coefficients obtained by applying <EQUATION> can be compressed. A quantisation vector of values, , is devised such that:

## Classification

There are several methods used to classify the features extracted from an EEG signal. Methods proposed by Wang et al. include Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA), Support Vector Machine (SVM) prediction, and *k*-Nearest Neighbours (kNN) algorithms < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/5205138>>. SVMs and LDA were observed to be widely used in classifiers

### Linear Discriminant Analysis (LDA)

Linear Discriminant Analysis (LDA) has historically been one of the most common classification techniques for BCIs. LDA aims to find a linear combination of features that can best separate two or more classes < <https://www.researchgate.net/publication/320367565_Comparison_of_the_EEG_Signal_Classifiers_LDA_NBC_and_GNBC_Based_on_Time-Frequency_Features>>. Following training of the classifier on a training data set, a set of eigenvectors representing each class is obtained. When a test observation is to be classified, the linear combination of features developed during testing is used to generate an eigenvector representative of the observation. The classifier then selects the class that produces the minimum Euclidean distance between the class eigenvector and the observation eigenvector. < <https://ieeexplore.ieee.org/document/7095376>>. As LDA is a machine learning approach, LDA classifiers can suffer from overfitting especially when the training dataset is small. However, it is proposed by Pang et al. that regularising the data by unsupervised clustering can reduce the likelihood of overfitting. < <https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=6799229>>. Similarly, since this approach is linear, LDA tends to not to suffer from overfitting as much as nonlinear classifiers < <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2914143/> >, but must operate under the assumption that the covariance of each class is the same < <http://mlweb.loria.fr/book/en/lda.html>>. Wang et al. achieved a classification accuracy of 82.86% on the 2003 Gratz BCI Competition Dataset III when classifying motor imagery EEG signals. The authors concluded that LDA, as a linear classifier is suitable for BCI applications due to its simplicity and stability < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/5205138>>.

### Quadratic Discriminant Analysis (QDA)

Quadratic Discriminant Analysis (QDA) operates in a similar manner to LDA, however removes the assumption that the covariance matrices of each class is the same < <https://onlinecourses.science.psu.edu/stat508/book/export/html/696>>. This removes the benefits discussed in Section 3.1 (resistance to overfitting) and introduces additional complexity. Wang et al. achieved a classification accuracy of 78.57% on the 2003 Gratz BCI Competition Dataset III when classifying motor imagery EEG signals < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/5205138>>. Thus, QDA is not a competitive classification solution in this application.

### Support Vector Machines (SVM)

Support Vector Machines (SVM) are another popular approach to EEG classification. These systems again employ machine learning, however in this case the goal is to define an *n*-1 dimensional plane in *n*-dimensional Euclidean space that divides the data classes, where the Euclidean distance between each class is maximised < <https://towardsdatascience.com/https-medium-com-pupalerushikesh-svm-f4b42800e989?gi=c212a83cdb57>>. This is achieved by first finding the support vectors. These are *n*-dimensional vectors defining the data points that are closest to the hyperplane (that is, hardest to classify). There is a theoretically infinite number of hyperplanes that can divide the classes <<http://people.csail.mit.edu/dsontag/courses/ml14/slides/lecture2.pdf>>, thus the SVM solves a trivial optimisation problem to determine the optimal plane. Once a new data point is supplied for classification, the dot product of the new data point’s vector and the plane is taken to determine what side of the plane it falls on. This is very computationally simple. The elegance of SVMs surrounds their implementation of the “kernel trick”, which enables non-linearly separable data to be separated by defining additional separation functions called kernels. These kernels can include polynomials, exponentials, and sigmoids < <http://web.mit.edu/6.034/wwwbob/svm-notes-long-08.pdf>>. Wang et al. achieved a classification accuracy of 82.86% on the 2003 Gratz BCI Competition Dataset III when classifying motor imagery EEG signals using a Linear SVM, and 84.29% using a Gaussian SVM. < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/5205138>>. Thus, the SVM is a viable solution to the EEG classification problem.

### *k*-Nearest Neighbours (kNN)

A *k*-Nearest Neighbours (kNN) algorithm establishes each training data point in an *n*-dimension parameter space, which can be defined by a vector of features (the features are obtained by the methods outlined in Section 2). When a new data point is to be classified, it is placed into the same parameter space. The data point is then classified by a voting system, where the class of the new data is the class of the majority of its “*k*-nearest neighbours”, or *k* closest training data points <<https://people.revoledu.com/kardi/tutorial/KNN/HowTo_KNN.html>>. There are several ways to calculate the “distance” between neighbouring data points. The most basic is Euclidean distance. However, measures such as cosine similarity, defined below, are also used in some applications (generally when the data to be classified is not sparse, creating large angular differences).

Where, is the similarity of data points and , and is the angle between data points and < <http://cs.carleton.edu/cs_comps/0910/netflixprize/final_results/knn/index.html>>.

This calculation is not computationally intensive in lower dimensions. However, the kNN algorithm suffers drastically in higher dimensions, as almost the entire parameter space is required to find the closest *k* data points, particularly if the data points are not close to each other <<http://www.cs.cornell.edu/courses/cs4780/2018fa/lectures/lecturenote02_kNN.html>>. This increases computation time significantly. Thus, in order to be implemented in an online BCI system, extensive (accurate) feature selection procedure is required.

Wang et al. achieved a classification accuracy of 84.29% on the 2003 Gratz BCI Competition Dataset III when classifying motor imagery EEG signals using a kNN approach. < <https://ieeexplore-ieee-org.ezproxy.newcastle.edu.au/document/5205138>>. Thus, the kNN approach is a viable solution to the EEG classification problem, however would require additional feature selection to be viable in an online classification setting.

## Simulation

Talk about results